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HIGH IMPEDANCE CATHODE FOLLOWERS FOR PIEZOELECTRIC GAGE RECORDING IN AIR BLAST RESEARCH

Bernard Soroka
Jacob Wenig

RDT & E Project No. 1M010501A006

BALLISTIC RESEARCH LABORATORIES

ABERDEEN PROVING GROUND, MARYLAND

B A L L I S T I C   R E S E A R C H   L A B O R A T O R I E S

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A B E R D E E N   P R O V I N G   G R O U N D,   M A R Y L A N D

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BSoroka/JWenig/jdk
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ABSTRACT

The design of four types of single tube cathode followers is described and selections made of the ones best suited for use with an eight-channel unit employed for air blast research.

Exact formulae as well as simplified good approximations for gain, input impedance and output impedance are derived in the Appendices. This report may serve as a reference source for cathode follower design and circuit analysis.

TABLE OF CONTENTS

LIST OF SYMBOLS

A = gain

C = capacitance in farads

E = voltage*

e_g = grid voltage

e_{in} = input voltage

e_{out} = output voltage

g_m = mutual transconductance - μ mho

g_p = plate conductance - μ mho

i = current*

I = external generator current

i_g = grid current

I_{in} = input current

i_{max} = maximum current

i_p = plate current

Q = charge in coulombs

R_g = grid resistance

R_{G_1} = grid resistance

R_{G_2} = grid resistance

R_{in} = input resistance

R_k = cathode resistance

R_{k_1} = upper cathode resistance

* All voltages in volts; all resistances in ohms; all current in amperes.

R_{k_2} = lower cathode resistance

R_{k_t} = total cathode resistance = $R_{k_1} + R_{k_2}$

R_{out} = output impedance

r_p = plate resistance

μ = amplification factor

ω = 2π times frequency

INTRODUCTION

During the course of investigating air blast phenomena and the reaction of structures to shock loading, piezoelectric transducers are used frequently for measuring the pressure-time histories of air blast waves. Since piezoelectric transducers are high impedance devices, a suitable impedance matching electronic circuit usually is required between the transducer and the relatively low input impedance recording system. The cathode follower is well suited to this task as it is fundamentally a high input - low output impedance device. This report analyzes four different types of single tube cathode followers, namely the simple cathode follower, the tapped cathode follower, the bypassed cathode follower and the bootstrapped cathode follower. Exact equations as well as simplified formulae for gain, input impedance and output impedance for these four types are derived in Appendix A and may serve as a reference source for cathode follower design. Gain, input impedance and output impedance are evaluated numerically for each.

CATHODE FOLLOWERS - GENERAL THEORY

The gain of a simple cathode follower (Fig. 1) is given as

$A = \frac{g_m \quad R_k}{1 + (g_m + g_p) R_k}$ where g_m is the mutual conductance of the tube, g_p is the plate conductance of the tube and R_k is the cathode resistance. As the cathode resistance is increased the gain of the cathode follower will increase. However, in this simple cathode follower circuit, larger values of cathode resistance change the tube operating point* resulting in a lower current in the tube and a corresponding reduction of g_m and g_p which also affect gain. Also, the input impedance of this cathode follower is quite low and equal to the grid resistance, R_g.

* Operating point is the static plate current and plate voltage of the tube without any external signal applied.

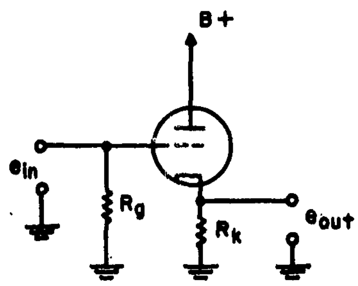

FIG. 1 SIMPLE CATHODE FOLLOWER

By using a tapped cathode follower (Fig. 2), these disadvantages are avoided.

In the tapped cathode follower (Fig. 2) R_{k_1} is selected to yield the proper operating point for the selected tube current and hence g_m and g_p remain the same. R_{k_2} is returned to the negative side of the power supply, resulting in a large value of R_k and hence larger gain. Larger values of R_{k_2} are thus possible but they again are limited, this time by the magnitude of the power supply it is feasible to use. For example, a 1 megohm cathode resistance (R_{k_2}) with 1 milliampere current would require B- to be 1000 volts. Returning the grid resistor R_g to the tap point, a, also has the effect of increasing the input resistance of the cathode follower.

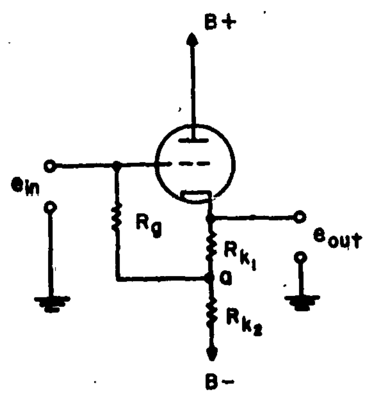

FIG. 2 TAPPED CATHODE FOLLOWER

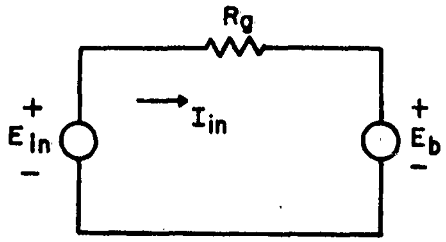

FIG. 3.

If we examine the simple circuit shown in Figure 3, we see that E_b provides a "bucking" voltage to the input voltage so that the net current flow $I_{in} = \dfrac{E_{in} - E_b}{R_g}$. Hence R_{in}, the resistance the input voltage "sees", is

$$R_{in} = \frac{E_{in}}{I_{in}} = \frac{E_{in}}{\dfrac{E_{in} - E_b}{R_g}} = R_g \frac{E_{in}}{E_{in} - E_b}$$

and the input resistance is, therefore, increased. The larger E_b becomes, the greater becomes the input impedance. This is exactly what occurs in the tapped cathode follower (Fig. 2). The voltage drop $I_p R_k$ is in the correct polarity to buck the input voltage. As derived in the Appendix,

$$R_{in} = \frac{R_g}{1-A\left[\dfrac{R_{k_2}}{R_{k_1} + R_{k_2}}\right]}$$

where A is the gain of the circuit at the cathode. This is known as partial bootstrapping. If R_g could be returned to a higher gain point eliminating the divider effect of R_{k_1} and R_{k_2}, a still higher input resistance could be obtained. The highest gain point in the cathode follower is directly at the cathode. Returning R_g to the cathode can be accomplished in two ways, one is by bypassing the cathode resistor with a large capacitor (Fig. 4), the second by using two grid resistors R_{g_1} and R_{g_2} and connecting the tap directly to the cathode (Fig. 5) through a capacitor.

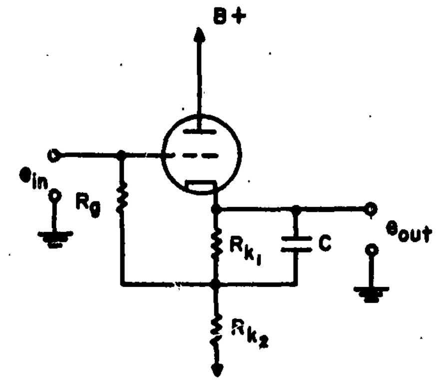

FIG. 4 BYPASSED CATHODE FOLLOWER

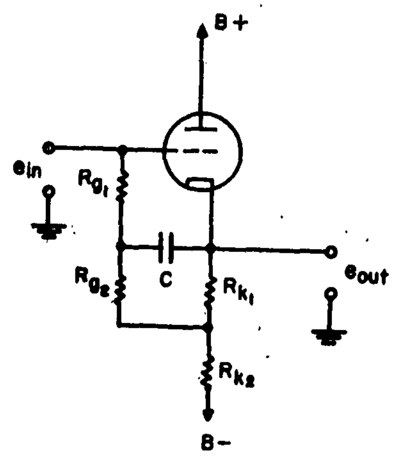

FIG. 5 BOOTSTRAPPED CATHODE FOLLOWER

The bypassed cathode is not a very satisfactory method since an extremely large bypass capacitor is required. The second method called the bootstrap method is best and effectively returns R_{g_1} to the cathode, signalwise, and shunts R_{k_1} with a large resistance R_{g_2}, so the resistance remains essentially R_{k_1}. The circuit of Fig. 5 is fully bootstrapped and for the values used in the cathode follower designed, gives approximately twice the input impedance of the tapped cathode follower. If now the gain could be made to approach unity, the input resistance would approach infinity, $R_{in} = \frac{R_g}{1 - A}$. However, the input resistance in the designed cathode follower would be high enough (1600 megohms) for piezoelectric gages. Higher input resistance (and higher gain) could be obtained by using a pentode tube as a cathode load instead of R_{k_2}. Still higher gain and input resistance could be obtained by using a "unity gain" amplifier which uses more than one tube for each cathode follower. This of course would add to the complexity of the circuitry especially where multiple channel cathode followers are required.

ADVANTAGES OF RETURNING R_k TO NEGATIVE SUPPLY

There are numerous advantages in returning the cathode resistance to a negative supply rather than to ground. These advantages are listed below:

1. A higher value of cathode resistance results in a higher gain which approaches but can never exceed unity.

2. The circuit becomes more stable as it becomes a constant current device.

3. There is a higher multiplication of input impedance.

4. There is a larger voltage swing capability with smaller currents so that larger input signals can be followed.

5. Larger negative signals can be followed without cutting the cathode follower off as in the case of grounded cathode resistors.

6. The output approaches zero potential. Thus, there is a minimum voltage strain across the output coupling capacitor and therefore minimum leakage.

DRIVING CAPABILITIES

The current drawn by the cathode follower is very important in the following respects:

1. It controls the maximum signal voltage that can be followed for a given cathode resistance.

2. It determines the values of mutual conductance and plate conductance and hence gain.

3. It determines the maximum capacitance load the tube can handle and still deliver its output signal.

The method of determining the current requirements of the tube with capacitance load, C, is as follows:

The charge on the capacitor is $Q = CE$ and since C is constant

$$i = \frac{dQ}{dt} = C\frac{dE}{dt}.$$

If the signal voltage is a sine wave of magnitude $A \sin \omega t$

$$i = CA\omega \cos \omega t \text{ ....(1)}$$

The maximum value occurs where $\cos \omega t = 1$ or $i_{max} = CA\omega$ ... (2)

For a 10 volt signal with $C = 100\ \mu\mu f$ and $f = 100$ KC

$$i_{max} = 100 \times 10^{-12} \times 10 \times 2\pi\, 10^5 = 0.63 \text{ Ma.}$$

If i_{max} is comparable to the current drawn statically by the cathode follower, then a distorted output can be produced.

From equation (2) one can see that capacity, maximum signal amplitude and frequency directly affect the maximum current requirements and hence signal output. For a given cathode follower, one must consider the required frequency response, maximum signal output, and the capacity which is to be driven.

FORMULAE AND DERIVATIONS

Formulae are derived in the Appendix A for all four cases of cathode follower. These formulae for gain, input impedance and output impedance are derived as exact forms as well as simplified close approximations. They are listed below in tabular form.

1. Simple Cathode Follower - Exact Formulae

$$A = \frac{g_m R_k}{1 + (g_m + g_p) R_k}$$

$$R_{in} = R_g$$

$$R_{out} = \frac{R_k}{1 + (g_m + g_p) R_k}$$

2. Tapped Cathode Follower

Exact

$$A = \frac{g_m (R_g R_{k_t} + R_{k_1} R_{k_2}) + R_{k_2}}{R_{k_2} + R_g + (g_m + g_p)(R_g R_{k_t} + R_{k_1} R_{k_2})}$$

$$R_{in} = \frac{R_g \left[1 + (g_m + g_p)(R_{k_t}) + \frac{R_{k_1} R_{k_2}}{R_g} \right] + R_{k_2}}{1 + (g_m + g_p) R_{k_1} + g_p R_{k_2}}$$

Alternative Method -

$$R_{out} = \frac{R_g R_{k_t} + R_{k_1} R_{k_2}}{R_{k_2} + R_g + (g_m + g_p)(R_g R_{k_t} + R_{k_1} R_{k_2})}$$

Simplified

$$A = \frac{g_m R_{k_t}}{1 + (g_m + g_p) R_{k_t}}$$

$$R_{in} = \frac{R_g \left[1 + (g_m + g_p) R_{k_t} \right]}{1 + (g_m + g_p) R_{k_1} + g_p R_{k_2}}$$

$$R_{in} = \frac{R_g}{1 - A \left[\frac{R_{k_2}}{R_{k_1} + R_{k_2}} \right]}$$

$$R_{out} = \frac{R_{k_t}}{1 + (g_m + g_p) R_{k_t}}$$

3. Bypassed Cathode Follower

Results are the same as in the Bootstrapped case except the value of R_{k_2} is substituted for R_{k_t} in the Bootstrap Formulae.

4. Bootstrapped Cathode Follower

Exact

$$A = \frac{(g_m R_g + 1) R_{k_t}}{R_{k_t} + R_g \left[1 + (g_m + g_p) R_{k_t}\right]}$$

$$R_{in} = \frac{R_g \left[1 + (g_m + g_p) R_{k_t}\right] + R_{k_t}}{1 + g_p R_{k_t}}$$

Alternative Method -

$$R_{out} = \frac{R_{k_t}}{1 + \frac{R_{k_t}}{R_g} + (g_m + g_p) R_{k_t}}$$

Simplified

$$A = \frac{g_m R_{k_t}}{1 + (g_m + g_p) R_{k_t}}$$

$$R_{in} = \frac{R_g \left[1 + (g_m + g_p) R_{k_t}\right]}{1 + g_p R_{k_t}}$$

$$R_{in} = \frac{R_g}{1-A}$$

$$R_{out} = \frac{R_{k_t}}{1 + (g_m + g_p) R_{k_t}}$$

The simplified formulae give answers within 0.1% (shown in Appendix B) of those obtained with the exact formulae and usually are used.

TUBE SELECTION AND DESIGN CHARACTERISTICS

Since the gain is essentially given by $A = \frac{g_m R_{k_t}}{1 + (g_m + g_p) R_{k_t}}$ it is desirable to select a tube with a high g_m for a reasonable cathode current. It is also a good procedure to select a bias voltage that is not too close to zero to avoid grid current effects. The tube selected for this case was a 12AX7 having a bias of -2 volts at a current of 1 milliampere. The tube

12AX7 PLATE CHARACTERISTICS

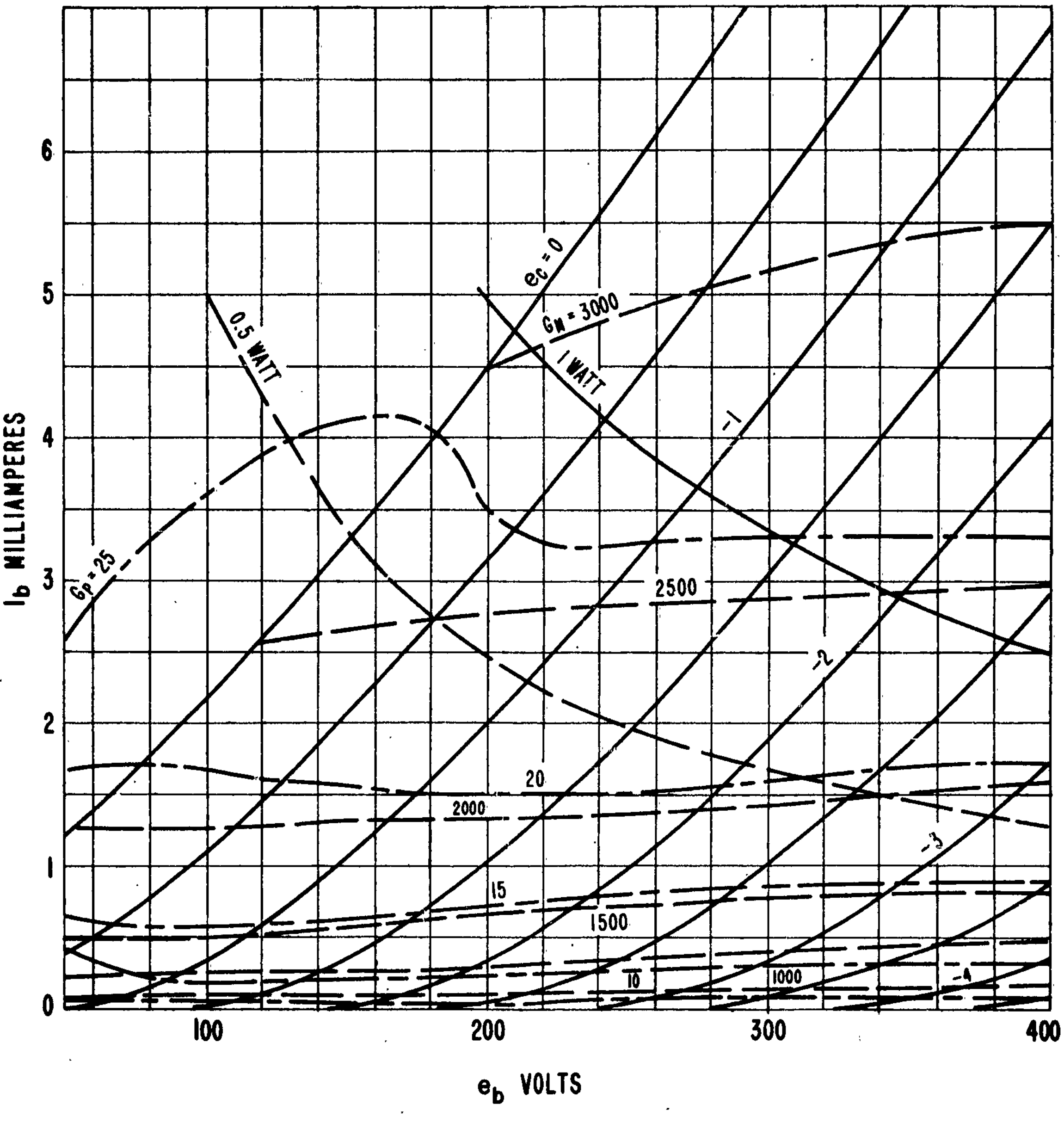

FIG. 6.

characteristics are shown in Figure 6*. A grid resistance of 22 megohms was used as a compromise which gave very little bias shift due to grid current. Much higher grid resistances could result in unstable operation due to shift of operating point.

A high quality, low leakage ceramic tube socket was used to avoid changes in grid resistance due to leakage resistance.

The output coupling capacitor was a 25 microfarad high quality Mylar unit needed to give a good low frequency response with a minimum of leakage and drift. Minimum leakage is very desirable as the cathode followers, in this case, are coupled to high gain DC amplifiers. Any leakage through the capacitor would cause the output signal of the DC amplifier to drift.

Design procedure for all cases and a composite table are shown in the next section.

DESIGN PROCEDURE FOR CATHODE FOLLOWER WITH TAPPED CATHODE

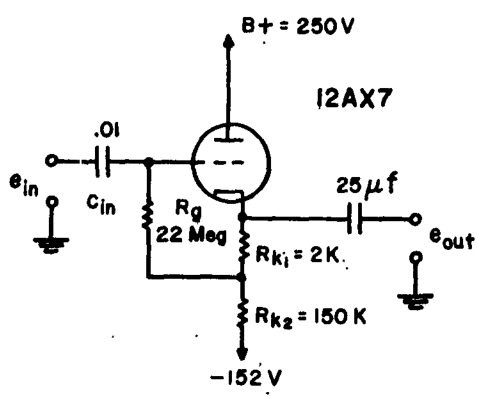

FIG. 7 TAPPED CATHODE FOLLOWER

* Curve taken from "Conductance Curve Design Manual" by Keats A. Pullen, Jr.

In the design procedure a selection was made for a B+ of 250 volts and e_c of -2 volts. It can then be shown from the tube characteristic curves of Fig. 6 that for e_p equal to 250 volts

$$i_p = 1 \text{ ma}$$

$$g_m = 1700\ \mu \text{ mhos}$$

$$g_p = 17\ \mu \text{ mhos}$$

Evaluating R_{k_1} which equals $\frac{e_c}{i_p}$ gives a value of 2000 ohms.

Now values for R_{k_2} of 150 K ohms and R_g of 22 megohms are selected. A negative supply to give 1 ma through $R_{k_1} + R_{k_2}$ which totals 152 K ohms is required thus yielding a negative supply of -152 volts.

Evaluating gain, R_{in} and R_{out} from the simplified equations for the tapped cathode follower give:

$$\text{Gain} = 0.986$$

$$R_{in} = 825 \text{ megohms}$$

$$R_{out} = 580 \text{ ohms}$$

The resultant input time constant is $0.01 \times 10^{-6} \times 825 \times 10^{6} = 8.2$ seconds. Similar results for the other three cases are shown in Table I.

DESCRIPTION OF 8 CHANNEL CATHODE FOLLOWER

Two designs were used in constructing an eight channel cathode follower. The simple cathode follower was ruled out as having too low an input resistance, i.e., 22 megohms, and the bypassed cathode follower was eliminated since it required an unreasonably high bypass capacitor. Thus, only the tapped and bootstrapped cathode followers were considered.

In the unit constructed, the cathode followers and power supply are contained on a single rack mounting chassis. Each channel is separately shielded and only one half of each 12AX7 tube is used to prevent cross talk or interference. The system is powered from a 115 Volt 60 cycle source so

TABLE I

COMPOSITE RESULTS FOR FOUR TYPES OF CATHODE FOLLOWERS

	Simple Cathode Follower	Tapped Cathode Follower	Bypassed Cathode Follower	Bootstrapped Cathode Follower
Gain	0.767	0.986	0.986	0.986
R_{in}	22 Megs.	825 Megs.	1600 Megs.	1600 Megs.
R_{out}	451 ohms	580 ohms	582 ohms	421 ohms
R_g	22 Megs.	22 Megs.	22 Megs.	22 Megs.
R_{k_1}	2 K	2 K	2 K	2 K
R_{k_2}	0	150 K	150 K	150 K
Input Time Constant	0.22 seconds	8.2 seconds	16 seconds	16 seconds

that a minimum of maintenance is required for field programs as against a battery operated system which requires constant attention. A circuit diagram of a power supply and a tapped cathode follower design is shown in Figure 8.

The power supply consists of an electronically regulated system to provide B+ voltages for the plates of the cathode follower tubes as well as DC power for the tube filaments. The tubes, which are rated at 12.6 Volts, are operated at 12 Volts to keep the grid current down and input impedance up. The regulated voltage on plates and filaments insures a minimum drift of tube characteristics.

The tapped cathode resistor is returned to a -150 Volt supply, regulated by a VR 150 tube. The proper values of R_{k_1} are selected to give proper grid bias and hence operating point of the cathode follower.

The bootstrapped cathode follower, (Fig. 5) using the identical power supply, was also used successfully.

The bootstrapped cathode follower gave about twice the input impedance as the tapped cathode follower. The gain was the same in both cases, but output impedance was somewhat lower for the bootstrapped version.

Good low frequency response is necessary for long duration measurements of pressure-time histories as characterized by large explosive charges.

CONCLUSION

The design equations for Gain, R_{in} and R_{out} gave results which agreed with experimental test within 10%. The single tube tapped cathode follower and bootstrapped cathode follower give input resistances of approximately 800 megohms and 1600 megohms respectively with a gain of 0.986. This is quite satisfactory for most requirements of high input impedance. The bootstrapped cathode follower is best as it gives the highest input impedance with no added circuitry complications except for the bootstrap capacitor.

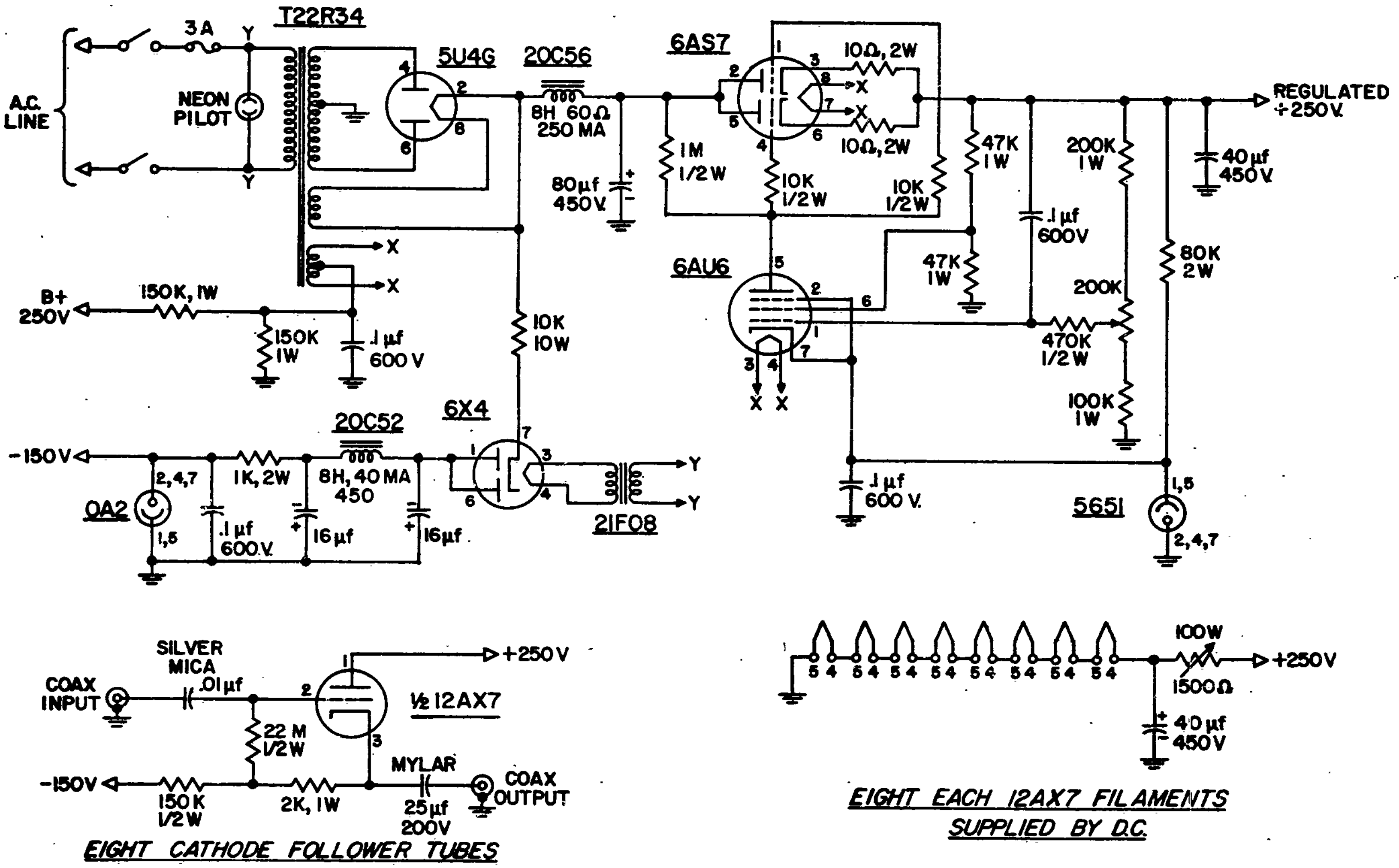

FIG. 8 – EIGHT CHANNEL CATHODE FOLLOWER

The eight channel cathode follower so designed has been satisfactorily used in the recording of blast pressures on piezoelectric gages with a multiple channel oscillograph recorder. It is characterized by simplicity of operation and reliable performance as well as a minimum of maintenance.

Bernard Soroka

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APPENDICES

A. Derivations of Gain, R_{in} and R_{out} for various cathode followers

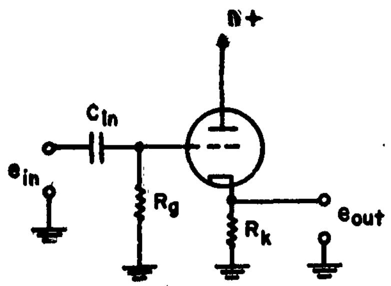
1. Simple Cathode Follower
2. Tapped Cathode Follower
3. Bypassed Cathode Follower
4. Bootstrapped Cathode Follower

B. Evaluation of Gain, R_{in} and R_{out} for the four cases of Cathode followers

APPENDIX A

1. Simple Cathode Follower

a. Gain



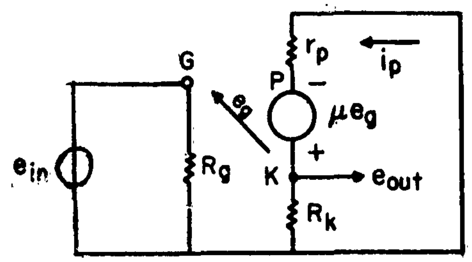


FIG. 9. SIMPLE CATHODE FOLLOWER - GAIN

Setting up the Mesh Equations for the Equivalent Circuit

$$\mu e_g - i_p (r_p + R_k) = 0 \quad \text{..............................(1)}$$

$$e_g = e_{in} - i_p R_k \quad \text{......................................(2)}$$

$$i_p = \frac{\mu e_{in}}{r_p + R_k (\mu + 1)} \qquad e_{out} = i_p R_k = \frac{\mu e_{in} R_k}{r_p + R_k (\mu + 1)}$$

$$\frac{e_{out}}{e_{in}} = A = \frac{\mu R_k}{r_p + R_k (\mu + 1)}, \qquad \mu = \frac{g_m}{g_p} \qquad r_p = \frac{1}{g_p}$$

$$A = \frac{g_m R_k}{1 + (g_m + g_p) R_k}$$

b. $R_{input} = R_g$

c. Output Impedance

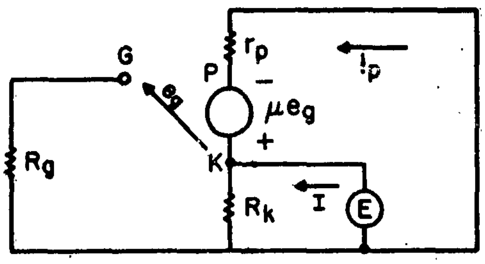


FIG. 10. SIMPLE CATHODE FOLLOWER – OUTPUT IMPEDANCE

$$\mu e_g - i_p (r_p + R_k) - IR_k = 0 \quad \ldots\ldots (3)$$

$$e_g = -(I + i_p) R_k \quad \ldots\ldots (4)$$

$$E - (I + i_p) R_k = 0 \quad \ldots\ldots (5)$$

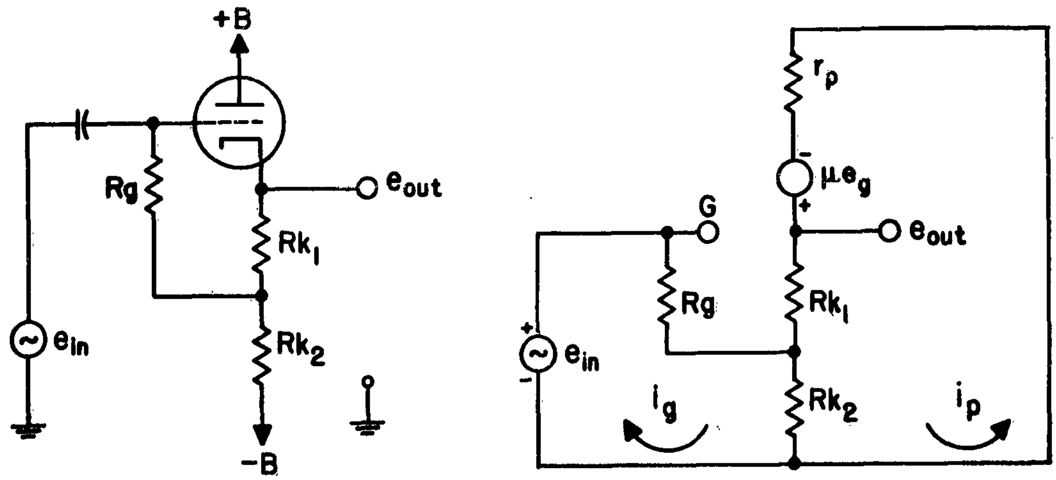
$$0 = i_p \left[r_p + R_k (\mu + 1) \right] + IR_k(\mu + 1) \quad \ldots\ldots (6)$$

$$E = i_p R_k + IR_k \quad \ldots\ldots (7)$$

Solving equations (6) and (7):

$$\frac{E}{I} = R_{out} = \frac{R_k r_p}{r_p + (\mu + 1) R_k} = \frac{R_k}{1 + (g_m + g_p)R_k}$$

2. Tapped Cathode Follower



CATHODE FOLLOWER CIRCUIT

EQUIVALENT CIRCUIT

FIG. 11 - GAIN AND INPUT IMPEDANCE

a. Gain

The mesh equations for the equivalent circuit are:

$$e_g = e_{in} - i_p (R_{k1} + R_{k2}) - i_g R_{k2} \text{ ....(8)}$$

$$\mu e_g - i_p (r_p + R_{k_1} + R_{k2}) - i_g R_{k_2} = 0 \text{ ....(9)}$$

$$e_{in} - i_g (R_g + R_{k2}) - i_p R_{k_2} = 0 \text{ ....(10)}$$

$$e_{out} = i_p (R_{k_1} + R_{k2}) + i_g R_{k2} \text{ ....(11)}$$

$$\mu e_{in} = i_p \left[r_p + (R_{k_1} + R_{k_2})\ (\mu + 1)\right] + i_g R_{k_2}\ (\mu + 1) \ldots\ldots (12)$$

$$e_{in} = i_p R_{k_2} \qquad\qquad + i_g\ (R_g + R_{k_2}) \ldots\ldots (13)$$

solving equations 12 and 13:

$$e_{out} = i_p\ (R_{k_1} + R_{k_2}) + i_g\ R_{k_2}$$

$$A = \frac{\mu R_g\ (R_{k_1} + R_{k_2}) + r_p\ R_{k_2} + R_{k_1} R_{k_2}\ \mu}{R_g \left[r_p + (R_{k_1} + R_{k_2})\ (\mu + 1)\right] + R_{k_2}\ r_p + R_{k_1} R_{k_2}\ (\mu + 1)}$$

$$\mu = g_m/g_p \qquad r_p = 1/g_p$$

$$A = \frac{g_m\ (R_g R_{k_t} + R_{k_1} R_{k_2}) + R_{k_2}}{R_g + R_{k_2} + (g_m + g_p)\ (R_g R_{k_t} + R_{k_1} R_{k_2})}$$

R_{k_t} = total cathode resistance = $R_{k_1} + R_{k_2}$

<u>Simple Form:</u>

if $R_g > R_{k_2}$

$$A = \frac{g_m\ (R_{k_1} + R_{k_2})}{1 + (g_m + g_p)\ (R_{k_1} + R_{k_2})} = \frac{g_m R_{k_t}}{1 + (g_m + g_p)\ R_{k_t}}$$

b. <u>Input Impedance</u>

Solving equations 12 and 13:

$$i_g = \frac{e_{in} \left[r_p + (R_{k_1} + R_{k_2})\ (\mu + 1)\right] - \mu e_{in} R_{k_2}}{(R_g + R_{k_2}) \left[r_p + (R_{k_1} + R_{k_2})\ (\mu + 1)\right] - R_{k_2}^{\ 2}\ (\mu + 1)}$$

$$R_{in} = \frac{e_{in}}{i_g} = \frac{R_g \left[r_p + (R_{k_1} + R_{k_2})\ (\mu + 1)\right] + R_{k_2} r_p + R_{k_1} R_{k_2}\ (\mu + 1)}{r_p + R_{k_1}\ (\mu + 1) + R_{k_2}}$$

$$\mu = g_m/g_p \quad r_p = 1/g_p$$

$$R_{in} = \frac{R_g \left[1 + (g_m + g_p)\, R_{k_t} + R_{k_1} \;\; R_{k2}/R_g \right] + R_{k_2}}{1 + (g_m + g_p)\, R_{k_1} + g_p\, R_{k2}}$$

Simple Form:

$R_g > R_{k2}$

$$R_{in} = \frac{R_g \left[1 + (g_m + g_p)\, (R_{k_t}) \right]}{1 + (g_m + g_p)\, R_{k_1} + g_p\, R_{k2}}$$

c. Simplified Version of Input Impedance

The input impedance can also be found in a simplified version.

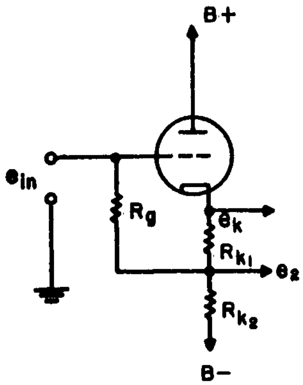


FIG. 12 SIMPLIFIED VERSION OF INPUT IMPEDANCE

$$R_{in} = \frac{E}{I} = \frac{e_{in}}{\frac{e_{in} - e_2}{R_g}} \quad \ldots\ldots\ldots \quad (14)$$

$$R_{in} = \frac{e_{in}}{e_{in} - e_2} \; R_g \quad \ldots\ldots\ldots \quad (15)$$

$$\frac{e_2}{e_k} = \frac{R_{k2}}{R_{k_1} + R_{k_2}} \quad \ldots\ldots\ldots \quad (16)$$

$$e_k = e_2 \; \frac{R_{k_1} + R_{k_2}}{R_{k_2}} \quad \ldots\ldots\ldots \quad (17)$$

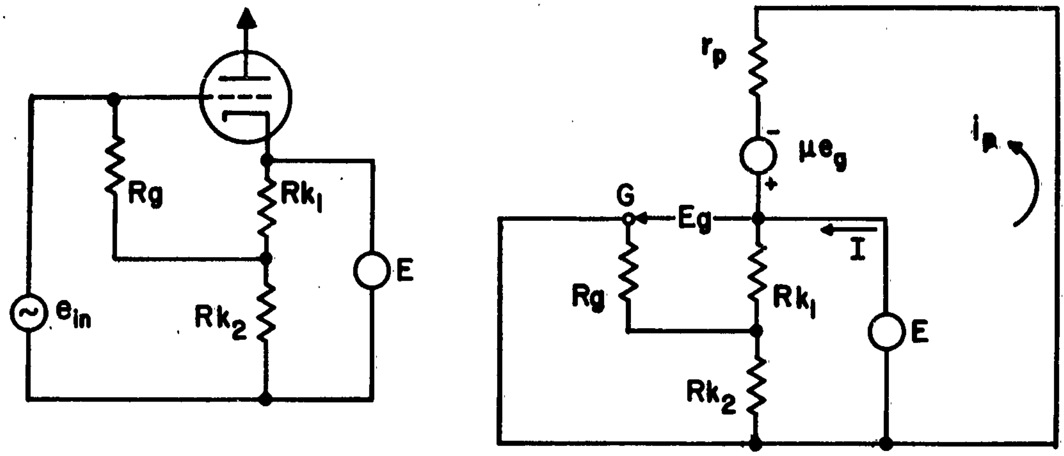
$$\text{Gain} = A = \frac{e_k}{e_{in}} \quad \ldots\ldots\ldots \quad (18)$$

Combining (15) and (16) $\quad A = \dfrac{e_2}{e_{in}} \; \dfrac{R_{k_1} + R_{k_2}}{R_{k_2}}$ $\quad \ldots\ldots\ldots \quad (19)$

$$\frac{e_2}{e_{in}} = A \; \frac{R_{k_2}}{R_{k_1} + R_{k_2}} \quad \ldots\ldots\ldots \quad (20)$$

$$R_{in} = R_g \; \frac{e_{in}}{e_{in} - e_2} = \frac{R_g}{1 - \frac{e_2}{e_{in}}} = \frac{R_g}{1 - A \left[\frac{R_{k_2}}{R_{k_1} + R_{k_2}} \right]}$$

d. Output Impedance



CATHODE FOLLOWER CIRCUIT EQUIVALENT CIRCUIT

FIG. 13 - OUTPUT IMPEDANCE

$$\mu\, e_g - (i_p + I)\, R_{k_1} - (i_p + I)\, \frac{R_{k2}\ R_g}{R_{k2} + R_g} - i_p\, r_p = 0$$

$$- e_g = E = (i_p + I) R_{k_1} + (i_p + I)\, \frac{R_{k2}\ R_g}{R_{k_2} + R_g}$$

$$- \mu\, E = i_p \left[r_p + R_{k_1} + \frac{R_{k_2}\ R_g}{R_{k_2} + R_g} \right] + I \left[R_{k_1} + \frac{R_{k2}\ R_g}{R_{k_2} + R_g} \right] \quad \ldots\ldots\ldots (21)$$

$$E = i_p \left[R_{k_1} + \frac{R_{k2}\ R_g}{R_{k_2}\ R_g} \right] + I \left[R_{k_1} + \frac{R_{k_2}\ R_g}{R_{k_2} + R_g} \right] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (22)$$

Solving equations 21 and 22:

$$R_{out} = \frac{E}{I} = \frac{r_p \left[R_{k_1} + \frac{R_{k_2} \; R_g}{R_{k_2} + R_g} \right]}{r_p + (\mu + 1) \left[R_{k_1} + \frac{R_{k_2} \; R_g}{R_{k_2} + R_g} \right]} \qquad \mu = g_m/g_p \qquad r_p = 1/g_p$$

$$R_{out} = \frac{R_g \; R_{k_t} + R_{k_1} \; R_{k_2}}{R_{k_2} + R_g + (g_m + g_p) \; (R_g \; R_{k_t} + R_{k_1} \; R_{k_2})}$$

Simplified Form:

if $R_g > R_{k_2}$

$$R_{out} = \frac{R_{k_1} + R_{k_2}}{1 + (g_m + g_p) \; (R_{k_1} + R_{k_2})} = \frac{R_{k_t}}{1 + (g_m + g_p) \; R_{k_t}}$$

3. Bypassed Cathode Follower

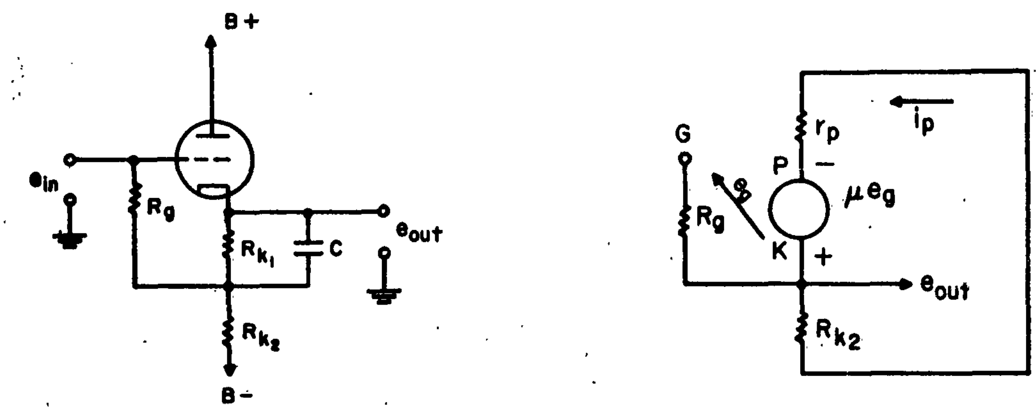


FIG. 14. BYPASSED CATHODE FOLLOWER EQUIVALENT CIRCUIT

With C large enough so its impedance is negligible at signal frequencies, R_{k_1} is effectively out of the circuit. Results are the same as in the Bootstrapped cathode follower except R_{k_2} is used in lieu of R_{k_t} in the Bootstrap formulae.

4. Bootstrapped Cathode Follower

a. Gain

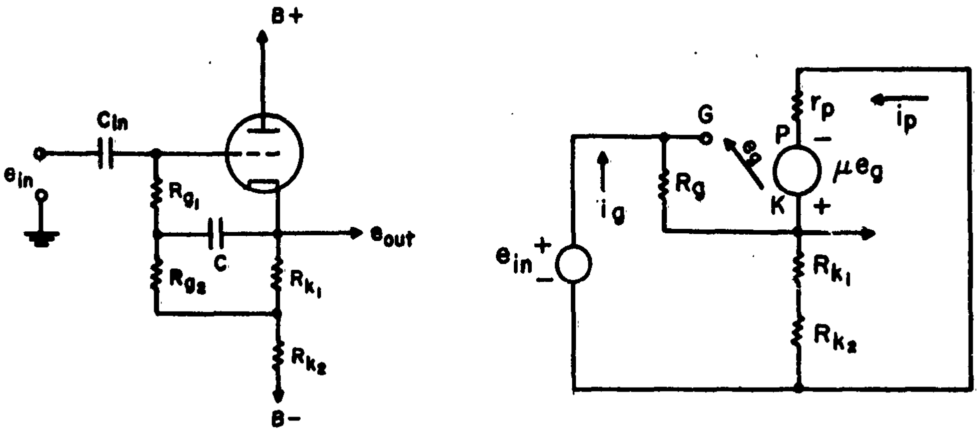


FIG. 15. BOOTSTRAPPED CATHODE FOLLOWER – GAIN

The mesh equations from the equivalent circuit are:

$$e_g = e_{in} - i_p\,(R_{k_1} + R_{k_2}) - i_g\,(R_{k_1} + R_{k_2}) \qquad (23)$$

$$\mu\, e_g - i_p\,(r_p + R_{k_1} + R_{k_2}) - i_g\,(R_{k_1} + R_{k_2}) = 0 \qquad (24)$$

$$e_{in} - i_g\,(R_g + R_{k_1} + R_{k_2}) - i_p\,(R_{k_1} + R_{k_2}) = 0 \qquad (25)$$

$$e_{out} = i_p\,(R_{k_1} + R_{k_2}) + i_g\,(R_{k_1} + R_{k_2}) \qquad (26)$$

$$\mu\, e_{in} = i_p\left[r_p + R_{k_1}(\mu + 1) + R_{k_2}(\mu + 1)\right] + i_g\left[R_{k_1}\,(\mu + 1) + R_{k_2}\,(\mu + 1)\right] \qquad (27)$$

$$e_{in} = i_p\,(R_{k_1} + R_{k_2}) + i_g\,(R_g + R_{k_1} + R_{k_2}) \qquad (28)$$

Solving equations 27 and 28:

$$e_{out} = (i_p + i_g)\ (R_{k_1} + R_{k_2})$$

$$A = \frac{e_{out}}{e_{in}} \quad \text{or}$$

$$A = \frac{(R_{k_1} + R_{k_2})\ (\mu\ R_g + r_p)}{R_g\ \left[r_p + (R_{k_1} + R_{k_2})\ (\mu + 1)\right] + (R_{k_1} + R_{k_2})\ r_p}$$

$$\mu = g_m/g_p \quad r_p = 1/g_p$$

$$A = \frac{(g_m\ R_g + 1)\ \ R_{k_t}}{R_g\ \left[1 + (g_m + g_p)\ R_{k_t}\right] + R_{k_t}}$$

Simplified Form:

if $g_m\ R_g \gg 1 \qquad R_g \gg R_{k_1} + R_{k_2}$

$$A = \frac{g_m\ \ R_{k_t}}{1 + (g_m + g_p)\ \ R_{k_t}}$$

b. Input Impedance

$$R_{in} = \frac{e_{in}}{i_g} = \frac{r_p\ (R_g + R_{k_1} + R_{k_2}) + R_g\ (\mu + 1)\ (R_{k_1} + R_{k_2})}{r_p + R_{k_1} + R_{k_2}}$$

$$\mu = g_m/g_p$$

$$R_{in} = \frac{R_g\ \left[1 + (g_m + g_p)\ \ R_{k_t}\right] + R_{k_t}}{1 + g_p\ R_{k_t}}$$

Simplified Form:

if $R_g \gg R_{k_1} + R_{k_2}$

$$R_{in} = \frac{R_g \left[1 + (g_m + g_p)\ \ R_{k_t}\right]}{1 + g_p\ R_{k_t}}$$

c. Output Impedance

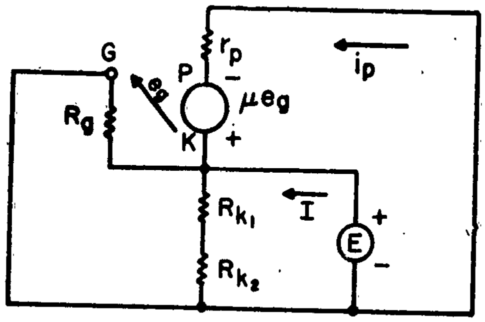


FIG. 16 . OUTPUT IMPEDANCE

let $R = R_g$ in parallel with $R_{k_1} + R_{k_2} = \dfrac{R_g\ (R_{k_1} + R_{k_2})}{R_g + R_{k_1} + R_{k_2}}$

Mesh equations:

$$\mu\, e_g - i_p\,(r_p + R) - I\,R = 0 \qquad (29)$$

$$- e_g = E = (i_p + I)\,R \qquad (30)$$

$$- \mu\, E = i_p\,(r_p + R) + I\,R \qquad (31)$$

$$E = i_p\,R + I\,R \qquad (32)$$

$$I = \frac{E\left[r_p + (\mu + 1)\,R\right]}{r_p\,R}$$

$$R_{out} = \frac{r_p\, R_g\, (R_{k_1} + R_{k_2})}{r_p\, (R_g + R_{k_1} + R_{k_2}) + (\mu + 1)\, r_g\, (R_{k_1} + R_{k_2})}$$

$$\mu = g_m/g_p \qquad r_p = 1/g_p$$

$$R_{out} = \frac{R_{k_t}}{1 + R_{k_t}/R_g + (g_m + g_p)\, R_{k_t}}$$

Simplified Form:

if $R_g \gg R_{k_1} + R_{k_2}$

$$R_{out} = \frac{R_{k_t}}{1 + (g_m + g_p)\, R_{k_t}}$$

APPENDIX B

Evaluation of Gain, Input Resistance and Output Resistance

Values for gain, input resistance and output resistance of the simple cathode follower, tapped cathode follower and bootstrapped cathode follower are evaluated for exact and simplified formulae. Great care is needed in evaluating R_{input} using the formula

$$R_{in} = \frac{R_g}{1-A} \quad \text{and} \quad \frac{R_g}{1-A\left[\dfrac{R_{k_2}}{R_{k_1} + R_{k_2}}\right]} \; .$$

Actually this is a very exact formula but if A is known to only 1 or 2 places, a large error is introduced in subtracting A from unity.

All values used in the equations are

$R_{k_1} = 2 \times 10^3$ ohms $\qquad R_{k_2} = 150 \times 10^3$ ohms $\qquad R_{k_t} = 152 \times 10^3$ ohms

$g_m = 1700 \times 10^{-6}$ mhos $\qquad g_p = 17 \times 10^{-6}$ ohms

$R_g = 22 \times 10^6$ ohms

Simple Cathode Follower

$$A = \frac{g_m R_k}{1 + (g_m + g_p) R_k} = 0.767$$

$$R_{in} = R_g = 22 \text{ megohms}$$

$$R_{out} = \frac{R_k}{1 + (g_m + g_p) R_k} = 451 \text{ ohms}$$

Tapped Cathode Follower

Exact

$$A = \frac{g_m (R_g R_{k_t} + R_{k_1} R_{k_2}) + R_{k_2}}{R_{k_2} + R_g + (g_m + g_p)(R_g R_{k_t} + R_{k_1} + R_{k_2})}$$

A = 0.986

$$R_{in} = \frac{R_g (1 + g_m + g_p) \frac{R_{k_1} R_{k_2}}{R_g} + R_{k_2}}{1 + (g_m + g_p) R_{k_1} + g_p R_{k_2}}$$

R_{in} = 825.2 megohms

$$R_{out} = \frac{R_g R_{k_t} + R_{k_1} R_{k_2}}{R_{k_2} + R_g + (g_m + g_p)(R_g R_{k_t} + R_{k_1} R_{k_2})}$$

R_{out} = 580.2 ohms

Simplified Form

$$A = \frac{g_m R_{k_t}}{1 + (g_m + g_p) R_{k_t}}$$

A = 0.986

$$R_{in} = \frac{R_g \left[1 + (g_m + g_p) R_{k_t}\right]}{1 + (g_m + g_p) R_{k_1} + g_p R_{k_2}}$$

R_{in} = 825.2 megohms

$$R_{in} = \frac{R_g}{1 - A \frac{R_{k_2}}{R_{k_1} + R_{k_2}}} = 821.5 \text{ megohms}$$

$$R_{out} = \frac{R_{k_t}}{1 + (g_m + g_p) R_{k_t}}$$

R_{out} = 580.2 ohms

Bootstrapped Cathode Follower

$$A = \frac{(g_m R_g + 1) R_{k_t}}{R_{k_t} + R_g + (g_m + g_p) R_g R_{k_t}}$$

A = 0.986

$$A = \frac{g_m R_{k_t}}{1 + (g_m + g_p) R_{k_t}}$$

A = 0.986

$$R_{in} = \frac{R_g \left[1 + (g_m + g_p)\ R_{k_t}\right] + R_{k_t}}{1 + g_p\ R_{k_t}}$$

R_{in} = 1608 megohms.

$$R_{in} = \frac{R_g \left[1 + (g_m + g_p)\ R_{k_t}\right]}{1 + g_p\ R_{k_t}}$$

R_{in} = 1608 megohms

Short method of evaluating R_{in}

$$R_{in} = \frac{R_g}{1-A} = 1606 \text{ megohms}$$

$$R_{out} = \frac{R_{k_t}}{1 + \frac{R_{k_t}}{R_g} + (g_m + g_p)R_{k_t}}$$

R_{out} = 580.2 ohms

$$R_{out} = \frac{R_{k_t}}{1 + (g_m + g_p)\ R_{k_t}}$$

R_{out} = 580.2 ohms

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Bernard Soroka and Jacob Wenig

BRL Memorandum Report No. 1474 April 1963

RDT & E Project No. 1M010501A006
UNCLASSIFIED Report

Blast gages - Recording devices
Air blast - Instrumentation
Piezo - electric gages

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Exact formulae as well as simplified good approximations for gain, input impedance and output impedance are derived in the Appendices. This report may serve as a reference source for cathode follower design and circuit analysis.

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